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COMPUTATIONAL RESULTS FOR WALBURN-SCHNECK AND CASSON MODELS FOR BLOOD IN SMALL VESSELS

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Abstract. *From the several constitutive models that have been proposed for blood, in this work we focus attention in the classical Casson model, considering an yield stress term, that have been adapted to allow hematocrit variation effects by Wang and Stoltz and the Walburn-Schneck four-parameters model that was developed based on an apparent viscosity consisting of two exponential functions of the hematocrit and of the protein content. In this work, blood flow behaviour is computationally investigated for the Walburn-Schneck and the Casson rheological models, in case of small vessels with non-uniform diameter. Computational results have been obtained here by a stabilized mixed finite element method in velocity and discontinuous pressure interpolations.*

keywords: *Blood Flow, Pseudoplastic Flow, Finite Element Method, Wlburn-Schneck Relation, Casson Relation*

1. Introduction

It is very well known that blood is a very complex fluid that alter its structure depending on the place it is passing along the circulatory loop. These differences come out depending on several factors, being very evident if blood is flowing in large or in small vessels. From the fluid mechanics point of view blood behaves in non-newtonian manner when going from large (in general Newtonian) to small vessels.

As a suspension of cells, in a nearly newtonian fluid called plasma with major part of the suspension in number being the red blood cell, it is commonly accepted that the aggregative phenomenon that happens at low shear rate combined with the rate of cells to whole blood (hematocrit) are two important factors to produce blood non-Newtonian behaviours.

Several constitutive equations have been proposed in order to describe the non-Newtonian behaviour of blood, none of them being the general one and experiments supply the constitutive parameters from viscometric flows only.

In order to investigate the flow characterization of two of those constitutive equations that were built from experimental viscometric data results, when they are applied to non viscometric flows, in this work computational experiments are performed for stenosed vessels in "meso-micro" circulation to the classical Casson relation compared with the Walburn-Schneck model considering the dependence of hematocrit and shear rate (through the complete deformation rate tensor) on the viscosity. Viscometric data have been used from the measurements of Wang and Stoltz, 1994 for the Casson model.

From the mathematical point of view, the nonlinearity and the divergence free internal restriction impose some care to be taken when approaching numerically these models. Computational results are obtained here by a mixed stabilized finite element method in continuous velocity and discontinuous pressure interpolations which generalizes the formulation of Karam and Loula, 1992 for non linear problems, allowing equal interpolation orders that are prohibited when classical methods are used.

2. Rheology of Blood in Brief

Blood is a concentrated suspension of several big and small components in an approximate newtonian fluid, the plasma. It can be divided as 55% vol. being plasma and 45% being cells or formed elements. Hematocrit is an important quantity defined as the volume rate of cells relative to the whole blood volume. Cells consist of approximately 95% of red blood cells - RBC - (without cell structure), 0.13% of white cells - WBC- and 4.5%

of platelets by number. Plasma is a dilute electrolytic solution with 8% of proteins in weight. These ingredients are sufficient to make blood a very complex structure that is able to adapt itself in a great extent depending on several circumstances to which it is submitted combined with the function it has to perform. This capability of adaptability reflects in altering the viscosity along the circulatory loop.

A general behaviour of blood in terms of its resistance to flow is characterized by a pseudoplastic effect when the shear rate is low, as is the case in small vessels, and a constant viscosity (Newtonian behaviour) when the shear rate increases, as is the general case in large vessels (macrocirculation). Blood rheology concentrates in the lower shear rate region where complexities, difficulties in measuring and modelling are challenging.

Some of the more important effects that can alter blood viscosity are briefly commented in the following.

2.1. Effect of temperature on the viscosity

There are many open questions in this item. But as the shear rate decreases more sensitive is the viscosity with the temperature. Viscosity decreasing inversely with temperature. This is important for filtration processes.

2.2. Effect of particle shape on the viscosity

Although the behaviour of the eritrocytes relative to the variability of the shape (as is the case inside the blood vessels) be very difficult to measure, it is known that in a suspension with rigid particles non-Newtonian behaviour increases when the particle shape is more different than the spherical one. For low shear rate, in experiments with red cells in Dextran 40 aqueous solution Whitmore, 1979 observed several non-Newtonian behaviours.

2.3. Effect of protein composition on the viscosity

Three of the numerous proteins are very important for the alteration of the viscosity. Fibrinogen, globulin and albumin. Globulin is responsible to augment blood viscosity and albumin tends to lower it. It is coherent with the known fact that albumin does not participate in the aggregation process rather than fibrinogen, globulin. If the only protein present is the globulin, viscosity increases relative to whole blood (only without fibrinogen) and it has a non-Newtonian behaviour by decreasing with increased shear rate and reaching a constant value when shear rate is high. If only fibrinogen is present it lowers the viscosity but it is constant even for low shear rate (newtonian behaviour). Fibrinogen and globulin participate in the clotting net process. Then fibrinogen is took out when making measurements. It is impossible to measure viscosity directly without breaking the clotting process. Then it is impossible to make direct experiments with whole blood.

2.4. Effect of hematocrit on the viscosity

Effect of hematocrit on the viscosity can be estimated through viscometric flows at constant shear rates. Viscosity increases linearly for high shear rate and high hematocrit. It is pseudoplastic when shear rate decreases and more pseudoplastic when hematocrit increases, in this case.

2.5. Effect of shear rate on the viscosity

Every effect is related to the shear rate. For low shear rate region, blood apparent viscosity increases when shear rate decreases. This is associated with two effects: (a) red cells of blood tend to aggregate forming *rouleaux* when shear rate decreases and aggregation is broken when shear rate increases, lowering again the viscosity; (b) for high shear rate, the flexible RBCs deform and tend to align with the flow direction, without aggregation, lowering the viscosity.

These are some of the effects that alter viscosity, and by them one can feel the difficulties in measuring apparent viscosity and mathematical modelling to characterize blood behaviour. Pathologies can alter the viscosity, as well.

To generate constitutive equations for fluids, one can measure the apparent viscosity in a rheometer (for blood, capillary or Couette type viscometers are most used) and then a curve fitting is performed. However, to measure the apparent viscosity, the shear rate has to be fixed with the fluid assumed to be Newtonian in each experiment. Moreover, it is difficult, or impossible, to make measurements and control all the variables that play a complex game. Then, taking some constitutive equation models, it is of interest to know how they behave and what are the differences among their results when they are applied to non-viscometric flows. This is important specially when in case of micro or non-macrocirculation, where shear rate is low and non-Newtonian behaviours are expected. For macrocirculation, Newtonian, linear and constant viscosity, is a suitable model since high shear rates are the case. Two of the above effects are focused here: hematocrit and shear rate.

3. Mathematical Model

For the purposes of these work, and considering small vessels where pulsating flow is negligible, we assume stationary creeping flow that can be characterized by solving the following system:

$$-\operatorname{div}\sigma = \mathbf{f} \tag{1}$$

$$\operatorname{div}\mathbf{u} = 0 \tag{2}$$

with the stress tensor

$$\sigma = -p\mathbf{I} + \tau, \tag{3}$$

where p is the hydrostatic pressure, $\mathbf{I}$ is the identity tensor, $\mathbf{f}$ denotes the body forces, $\mathbf{u}$ is the velocity field and τ is the nonlinear shear rate tensor coming from each of the following constitutive equations.

3.1. Casson Model

One of the first non linear constitutive equations to model blood is the classical Casson's equation, (Casson, 1959), originally conceived for paint suspension and firstly used by Scott-Blair, 1959 for blood

$$\tau^{1/2} = \tau_y^{1/2} + \eta|\gamma|^{1/2} \tag{4}$$

that predict an "yield stress", τ_y, and η being an asymptotic "Newtonian" value for high shear rate. It can be rewritten to allow hematocrit dependence as:

$$\tau(H)^{1/2} = \tau_y(H)^{1/2} + \eta(H)|\gamma|^{1/2} \tag{5}$$

And using the data measured by Wang and Stoltz, 1994 these functions become

$$\tau_y = (0.625H)^3 \tag{6}$$

$$\eta = \eta_P(1-H)^{-2.5}, \tag{7}$$

$$\eta_P = 1.22mPa\,s \qquad 0 < H < 1. \tag{8}$$

Without entering the discussion on the existence of an yield stress, for blood, we are considering this equation only as a two function parameters constitutive relation.

3.2. Walburn-Schneck Model

Starting from the power-law relation, given in a general form by

$$\tau = \mu_{app}\gamma^{n-1} \tag{9}$$

$$\mu_{app} = K|\gamma|^{n-1} \tag{10}$$

where μ_{app} is the apparent viscosity, K is the consistence parameter and n is the power index, that comes from the Cross model, (Cross, 1965), Walburn and Schneck, 1976 proposed an empirical equation, with four parameters, for the shear stress×shear rate relation, depending on the hematocrit, and, additionally, including a fixed amount of proteins:

$$\mu_{app} = C_1 \exp(100HC_2)\exp\left(\frac{C_4 TPMA}{(100H)^2}\right)\gamma^{-100HC_3} \tag{11}$$

with
$C_1 = 0.797mPa\,s$,
$C_2 = 0.608$,
$C_3 = 0.00499$,
$C_4 = 14.585lg^{-1}$,
$TPMA = 25.9gl^{-1}$.

and $TPMA$ being the amount of proteins except albumin. Basically it is composed of fibrinogen and globulin.

4. Finite element method used

By introducing each of the above models in the Eqs. (1) framework, the system has been solved here by applying the mixed stabilized finite element method in velocity-discontinuous pressure interpolations introduced in Bortoloti and Karam, 2004 which generalizes the formulation of Karam and Loula, 1992, for non linear problems, allowing equal interpolation orders and a Uzawa algorithm to solve for the nonlinearities.

5. Computational results

Numerical experiments have been performed for an axisymmetric 50% stenosed vessel whose mesh is illustrated in Fig. 1, with dimensions $d_1 = 500\mu m$, $d_2 = 50\mu m$. Meshes have been constructed with 2000 biquadratic elements. Hematocrits of 30% to 55% have been tested, and the range from 30% to 40% is considered as pathological. It was prescribed an inlet velocity of $250\mu m/s$. Results may be observed from Figs. 2(a)-4(f) in terms of pressure along the center axis ($d_1/2$), Fig. 2, velocity profiles, Fig. 3, and apparent viscosities, Fig. 4, at the center section of the stenose ($d_2/2$).

From the pressure results, in Fig 2, it can be seen that Casson's model gives lower pressure values along all the vessel, also giving a lower global pressure gradient. For very low hematocrit, 30%, it can be observed that Casson's model gives lower values than the Newtonian pressure profile, while Walburn-Schneck's gives higher values even in this case. By observing the stenosed area, the two models give almost the same pressure gradient for low hematocrits, but Walburn-Schneck's gives a higher gradient when the concentration increases Figs 2(e,f).

From the velocity profiles, it is very difficult to distinguish Casson results from the Newtonian behaviour, while the Walburn-Schneck's model varies from nearly Newtonian for low hematocrit to more non-Newtonian when the hematocrit increases.

From the apparent viscosity results, Fig. 4, it can be observed that for very low hematocrit, 30% and 35%, Casson's model is very close to the Newtonian behaviour while Walburn-Schneck's exhibit more differences, with higher values than those of the Casson's model along all the stenosed section. For medium hematocrit, 40% and 45%, the two models almost equal apparent viscosity magnitudes near the wall, but maintain the differences in the core of the section. Increasing the hematocrit, the two models invert places at the region near the wall, with Walburn-Schneck's giving lower apparent viscosity values than the Casson's, butwith even higher values in the core.

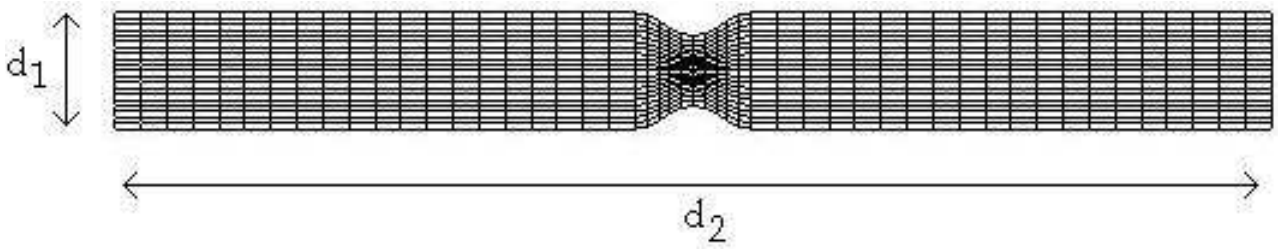

Figure 1: Stenotic mesh

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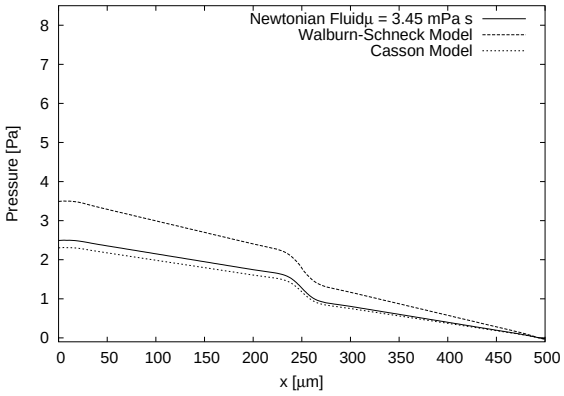
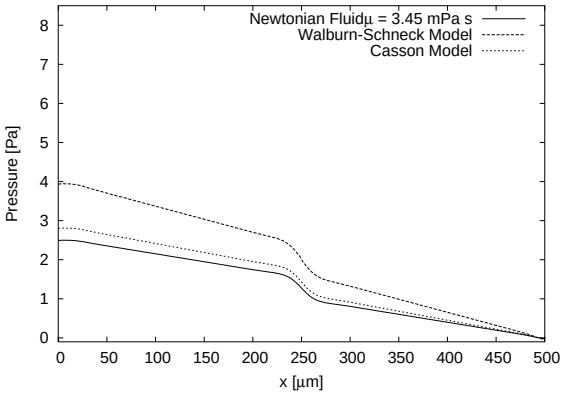
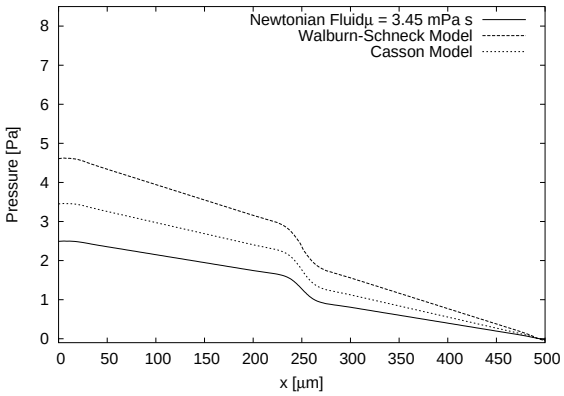


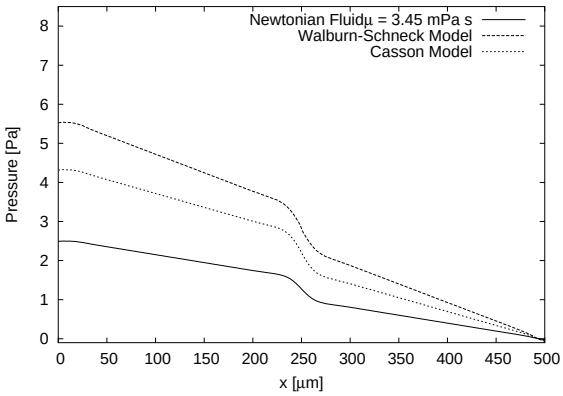
(a) H=0.3



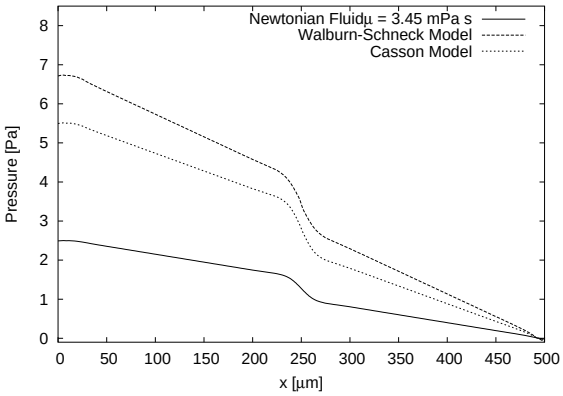
(b) H=0.35



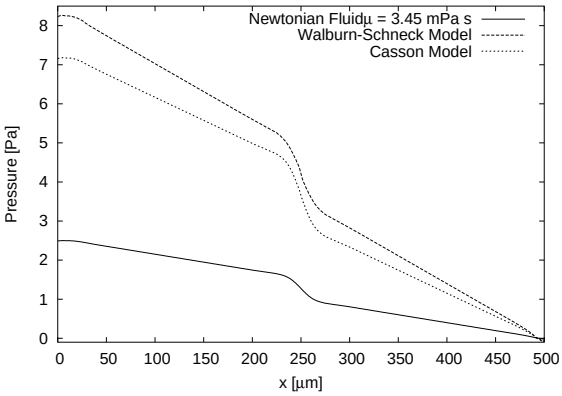
(c) H=0.4



(d) H=0.45

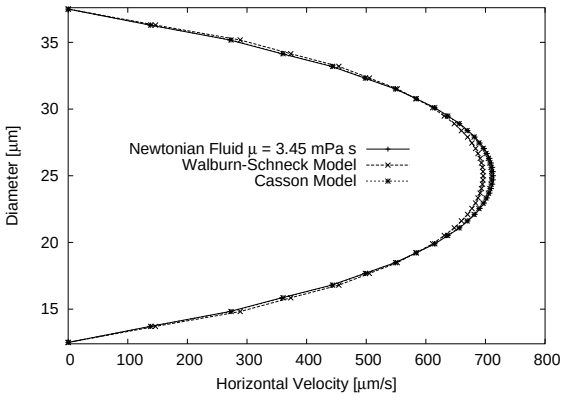


(e) H=0.5



(f) H=0.55

Figure 2: Pressures



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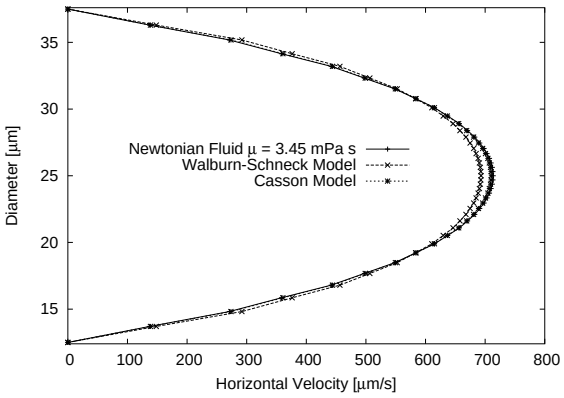
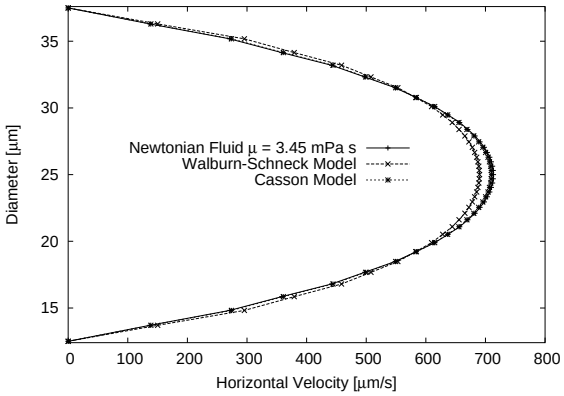


(b) H=0.35



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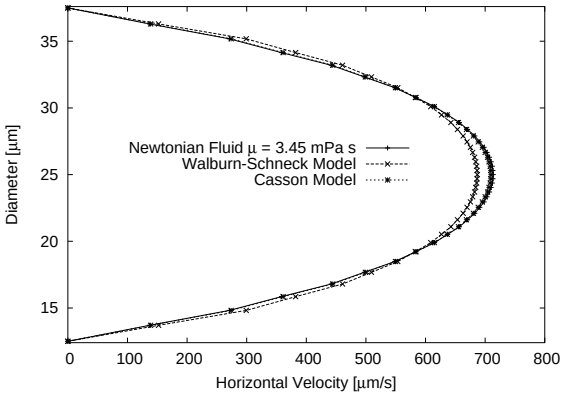

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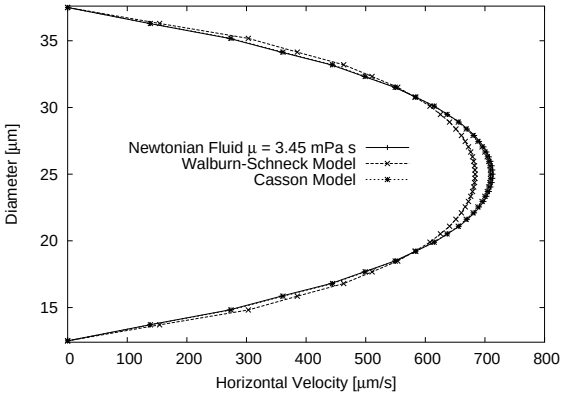

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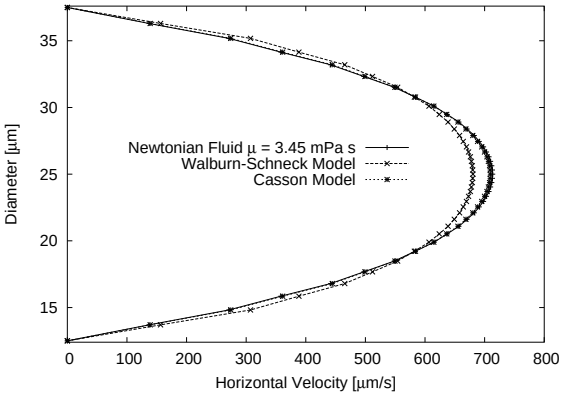
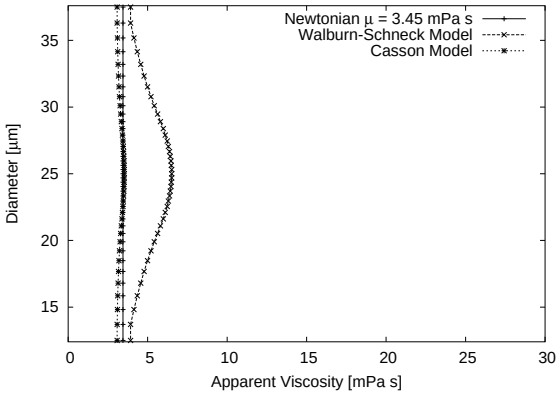


(f) H=0.55

Figure 3: Velocities



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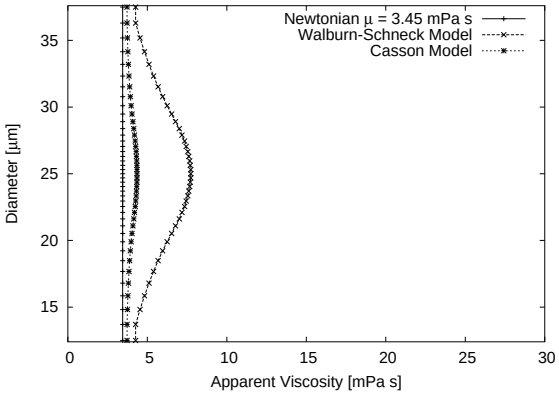

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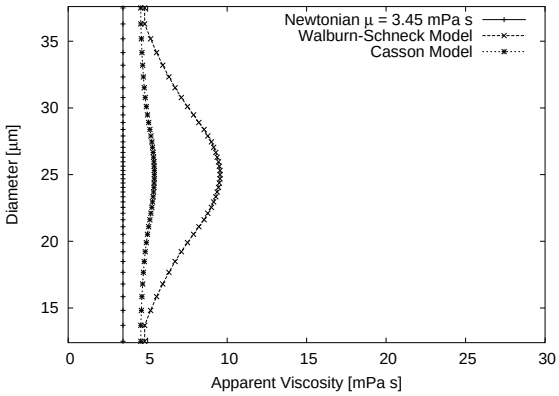
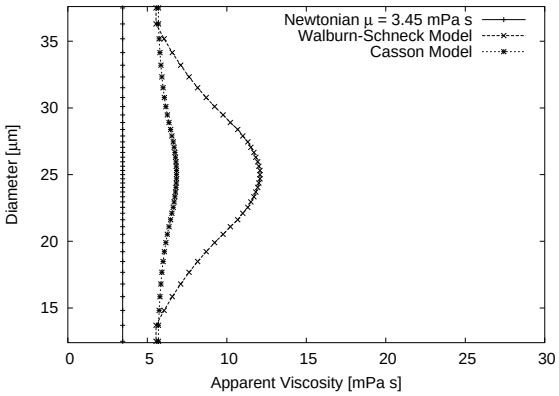


(c) H=0.4



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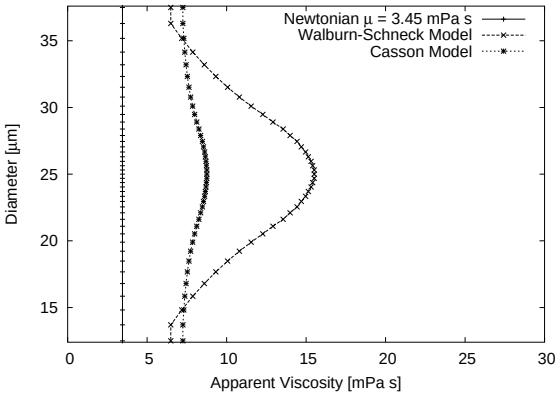

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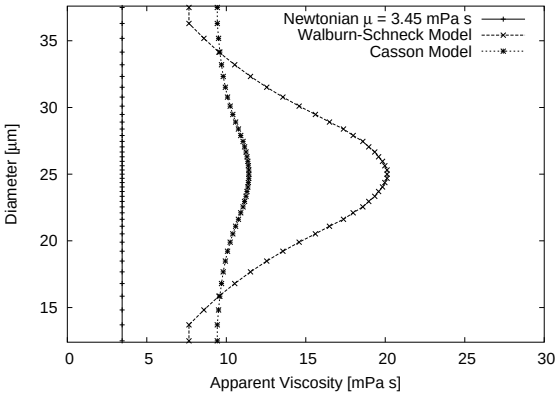

(f) H=0.55

Figure 4: Apparent viscosities

6. Conclusions and Remarks

In this work, some differences between Casson and Walburn-Schneck models models have been studied, considering the dependence of the apparent viscosity with hematocrit besides the shear rate for blood flow in small stenosed vessels for a simple non-viscometric case, for hematocrit ranging from 30% to 55%.

A mixed stabilized finite element method in velocity-discontinuous pressure has been used, allowing equal order interpolations and ensuring more accuracy once the constraint is locally satisfied.

The numerical results reveal different behaviours when Casson and Walburn-Schneck models are considered, with hematocrit and shear playing important roles. The Walburn-Schneck model shown to be more able to capture the non-Newtonian behaviour when increasing the hematocrit, even suggesting, indirectly from the apparent viscosity results, that a kind of sigma-effect is being captured in this case.

7. Acknowledgements

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